Critical analysis of the ATA formula

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Introduction

One step in the trunk-formula method, as presented by the Council of Tree and Landscape Appraisers (CTLA), involves measuring the trunk cross-sectional area of the tree at 54″ above grade. The 8th edition of the ***Guide for Plant Appraisal*** (1992) introduced the Adjusted Trunk Area (ATA)—an adjustment for tree trunks greater than 30" in diameter. Use of the ATA was continued in the 9th edition of the *Guide* (2000).

ATA was designed by CTLA "on the basis of the perceived increase in tree size, expected longevity, anticipated maintenance and structural safety." The Council's perception was that above 30 inches in diameter, "tree values based on trunk area became unrealistically high." Trunk area increases geometrically. CTLA did not believe that a geometric increase was realistic. ATA was introduced as a modification of the geometric increase resulting in a slower rate of trunk area. For example, the actual trunk area of a 40" diameter tree is 1,256 sq. in. The ATA is 1,149 sq. in.

Limitations of the current ATA Formula

The ATA formula has several weaknesses (**Fig. 1**). It is a downward-sloping quadratic equation that does not behave in the same way as the data it is intended to model: it has a theoretical maximum, and values for large trees decrease with size. The formula is also arbitrary in nature – it was chosen with an arbitrary starting point and an arbitrary data set to calculate the coefficients. In this research, I will illustrate the problems with this formula and propose several solutions.

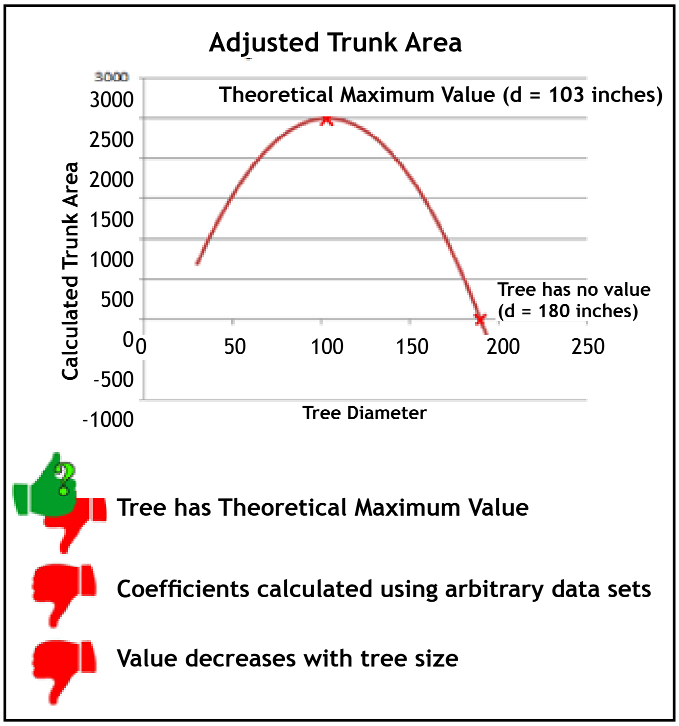


Figure 1: Adjusted Trunk Area curve showing the relationship of the tree diameter to the calculated trunk area using the formula. The maximum value and the zero-intercept are plotted to illustrate the shortcomings of the formula.

> *The key problem with the ATA formula is its arbitrary nature.*

CTLA arbitrarily adjusted the trunk area to fit a quadratic curve using linear regression (CTLA 2000). Outside the range of 30 to 103.4 inches in tree diameter, the formula is conceptually unsupported, thus demonstrating the arbitrary nature of the concept of ATA.

As the trunk area increases beyond 103.4 inches, the tree loses value due to the projection of the quadratic equation. In this case, ***even a tree in perfect health, location, and condition would still appraise for a lower value than a smaller comparable tree***. Further projection of this formula results in an ATA of 0 for a diameter of approximately 189.7 inches. The giant sequoia General Sherman and its neighbors would all appraise at a negative ATA because of their large diameters well over 189.7 inches.

In deriving these figures, we begin with the ATA formula as published in the Guide for Plant Appraisal:

$$ATA = -0.335d^2 + 69.3d - 1087$$

1) This formula is a downward-sloping quadratic equation. To find the diameter value that produces the maximum value of the ATA, I took the derivative of both sides of the equation.

$$\frac{d}{dd}\,ATA = \frac{d}{dd}\left(-\,0.335d^2 + 69.3\text{d} - 1087\right)$$

$$\frac{dATA}{dd} = -0.67d + 69.3$$

2) The maximum value occurs at the point where the rate of change of the formula is equal to 0.

$$0 = -\ 0.67d + 69.3$$

$$0.67d = 69.3$$

$$d \approx\ 103.4$$

3) The maximum ATA occurs when the subject tree is 103 inches in diameter at breast height. Beyond this point, the ATA decreases to 0, which implies that regardless of their condition, location, or species ratings, trees become less valuable as their trunks increase in diameter beyond 103 inches.

4) The 0 point can be found by using the quadratic equation and plugging in the coefficients from the ATA formula:

$$d = \frac{-69.3 \pm \sqrt{(69.3)^2 - 4(-0.335)(-1087)}}{2(-0.335)}$$

$$d \approx \frac{-69.3 \pm 57.84}{-0.67}$$

5) Because the ATA formula specifies that it only applies to diameters greater than 30 inches, the lower intercept may be ignored. Therefore, the diameter value at which the ATA equals 0 is:

$$d \approx 103.43 + 86.33$$

$$d \approx 189.76$$

6) All diameters greater than 189.76 inches will result in a negative ATA, which implies that regardless of the condition, species, or location ratings, the tree is completely valueless when it has a trunk diameter greater than 15 feet.

The problem with the ATA formula that this exercise illustrates is the poor choice of function. A downward sloping quadratic equation does not represent the intended change in value of a tree relative to its trunk area because it eventually slopes downward to 0, regardless of its coefficients.

Potential Solution 1: Logistic Regression (Maximum Tree Value)

One potential choice of formula may be a logistic function, forming an S-curve (Fig. 2). This function is designed to model an exponential increase within finite limits. The modified trunk area increases rapidly but then slows down and eventually comes very close to an asymptote, a maximum value for the function.

Here, I offer a possible solution by choosing the data set of trunk diameters from 1 to 50 and their respective actual trunk areas and fitting them to a logistic curve.

$$d = \{\ 1, 2, \cdots, 50\}$$

$$TA = \{\ 0.78, 3.14, \cdots, 1963.49\}$$

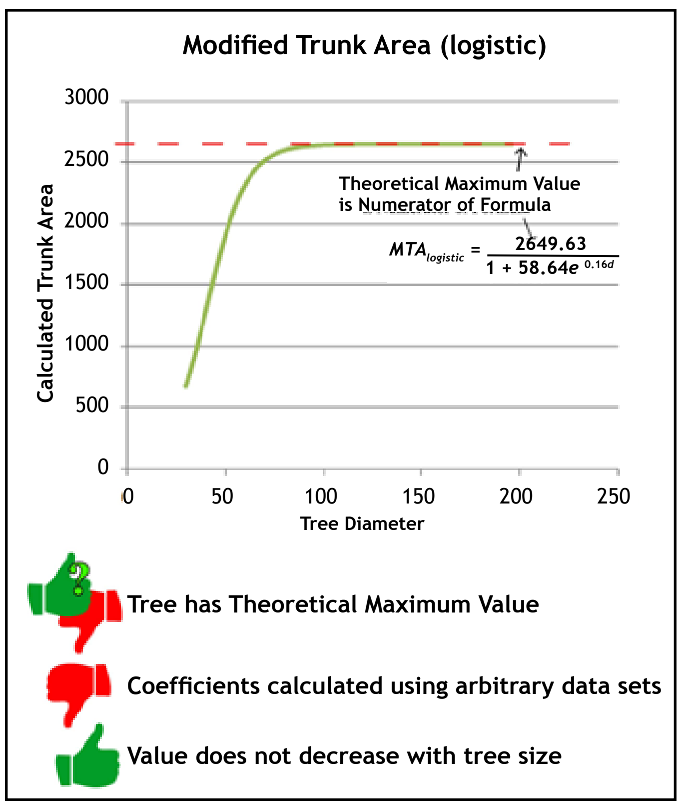
Logistic regression calculators are available with most spreadsheet software, and they calculate the coefficients of the formula that fits the data with the smallest possible error. Plugging the aforementioned data into a logistic regression calculator yields the following formula, which I have named the Modified Trunk Area (MTA):

$$MTA_{logistic} = \frac{2649.63}{1 + 58.64e^{-0.10d}}$$

There are two key disclaimers for this new, proposed logistic-regression MTA formula. First, the existence of an asymptote implies an assumption that there is a maximum value for a tree, regardless of its size. That assumption is based on the personal values of the authors of the formula, which is one of the weaknesses of using any formula calculation for valuing a tree. Unless it can be universally agreed by industry practitioners that this assumption is true, then this formula will ultimately be challenged in litigation or scholarly discussion.

Second, this theoretical maximum can be set at any value simply by choosing a different data set. For example,

Figure 2: Modified Trunk Area Logistic showing the relationship of tree diameter to the calculated trunk area using the first alternative formula proposed in the article. The asymptote is the numerator of the formula.



if the data set chosen was the set of diameters between 1 inch and 30 inches, the formula would be:

$$MTA_{logistic} = \frac{966.19}{1 + 58.54e^{-0.166d}}$$

The theoretical maximum trunk area value would only be 966 square inches. Likewise, if the data was the set of diameters between 1 and 100 inches, the theoretical maximum would be 10,491 square inches. The committee in charge of authoring this formula and choosing the appropriate data set would therefore have an opportunity to incorporate personal bias into the formula by choosing the "maximum valuable size" for a tree. Again, if this decision cannot be universally accepted, then it too will be challenged by practitioners.

Although the logistic curve more accurately represents the tapering of value that the CTLA seeks to model, it still contains the same potential for committee authorship bias as does the ATA formula.

Potential Solution 2: Logarithmic Regression (No Maximum Value)

A second possible alternative to a quadratic equation is a logarithmic equation. Like the logistic equation, the rate of value increase gradually tapers as values become larger. Unlike the logistic equation, there is no maximum value. This formula takes the form of:

$$MTA_{ln} = a\, ln(d) + b$$

The coefficients for this formula can be calculated by selecting a point on the unadjusted Trunk Area curve and calculating the slope of the line at that point. At a diameter of 30 inches (d = 30), the trunk area is:

$$TA = \frac{\pi}{4} d^2 = \frac{\pi}{4}(30)^2 = 225\pi$$

To find the slope of the line at this point, calculate the derivative of the equation for the area of a circle and plug in the value of d = 30:

$$\frac{dTA}{dd} = \frac{d}{dd}\left(\frac{\pi}{4} d^2\right) = \frac{\pi}{2} d$$

$$\frac{dTA}{dd} = \frac{\pi}{2}(30) = 15\pi$$

Then, calculate the derivative of the logarithmic equation and plug in the point and slope values to find the value of the first coefficient:

$$\frac{dMTA_{ln}}{dd} = \frac{d}{dd}\left(a\, ln\,(d) + b\right) = \frac{a}{d}$$

$$15\pi = \frac{a}{30}$$

$$a = 450\pi$$

Finally, plug in the point values and the first coefficient to calculate the second coefficient:

$$225\pi = 450\pi\, ln(30) + b$$

$$b = -4101$$

Thus yielding the formula for MTA_{Ln}:

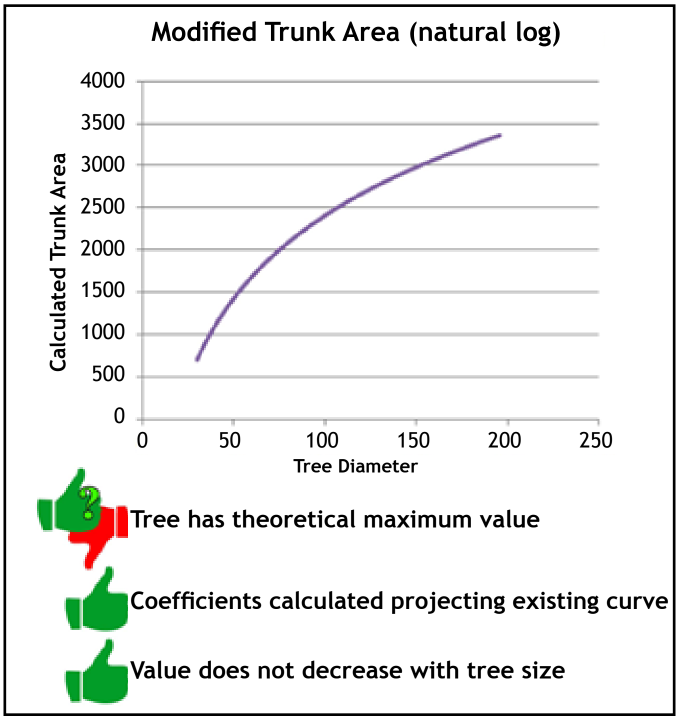
$$MTA_{ln} = 450\pi\, ln(d) - 4101$$

The primary advantage to this formula is it has no maximum value, so larger trees are still worth more. The MTA_{ln} formula tapers the value increase so that large trees do not increase in value too rapidly (Fig. 3). The coefficients are calculated by projecting the Trunk Area curve instead of using an arbitrary data set. However, just like the other formulas, the starting point is still arbitrary.

Potential Solution 3: Eliminate the ATA Formula

If ATA is eliminated, then the base cost will follow a standard quadratic curve. There will be no arbitrary adjust-

Figure 3: Modified Trunk Area $_{\text{Natural Log}}$ showing the relationship of tree diameter to the calculated trunk area using the second alternative formula proposed in the article. There is no maximum value, but it moderates its incremental rate of increase for larger trees.



ments made based on the opinions of the formula writers, and there will be no inconsistent extrapolations. Large trees will increase rapidly in base cost as they increase in size.

Many appraising arborists believe this to be a poor option for the formula because the resulting amount does not accurately reflect the value of the subject trees. The key element that is often forgotten in the debate over appraisal methodology is that the Trunk Formula Method was not intended to calculate a value of a tree – it was intended to calculate a cost solution. The cost solution is the result of extrapolating costs to recreate the subject tree. This solution will not necessarily equal the value created by the tree (**Fig. 4**).

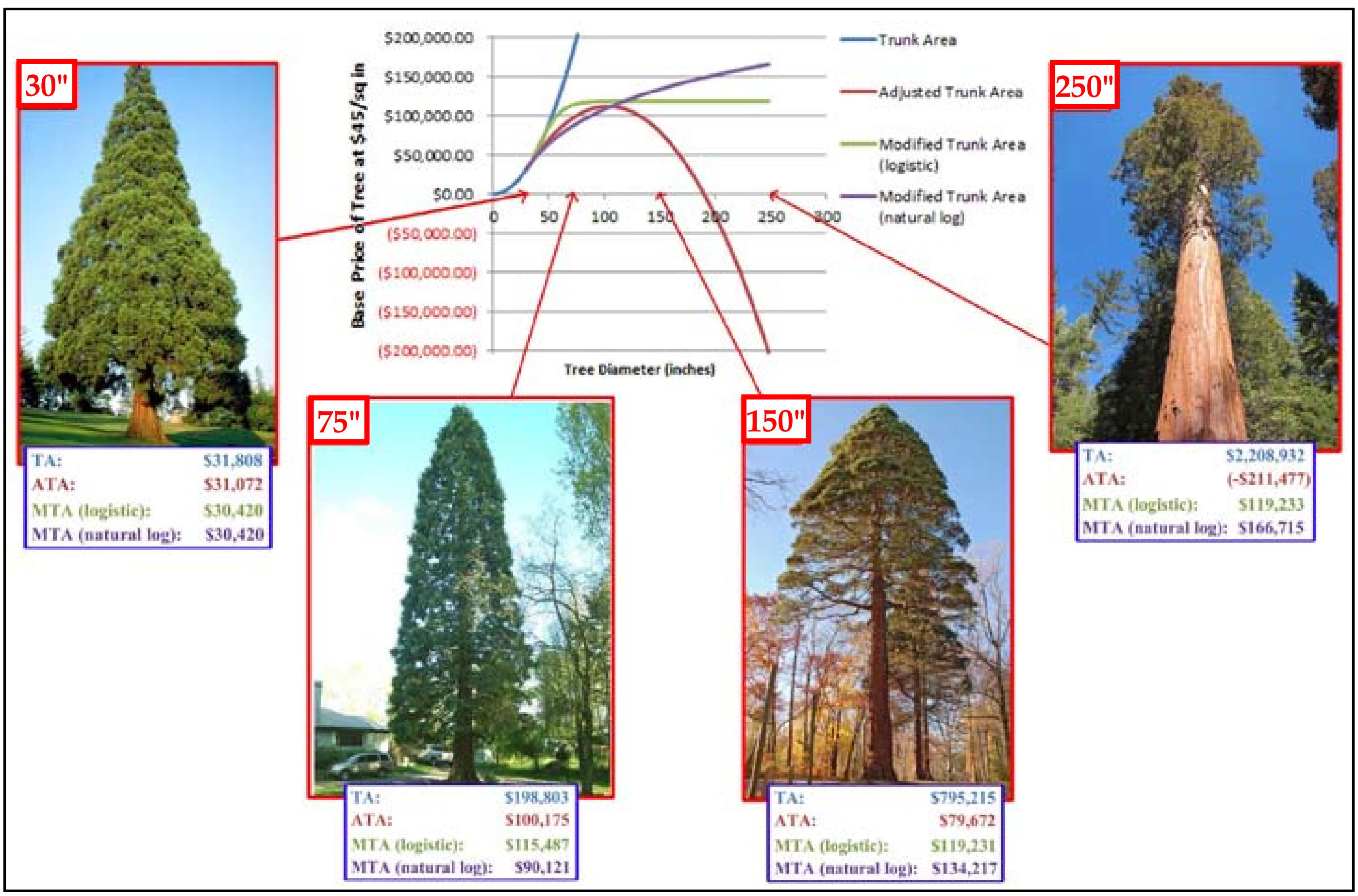
The Trunk Formula Method is a method within the cost approach of appraisal, intended to provide a cost for replacing the tree. Whether that cost solution accurately reflects the value of the tree is not an issue that should be decided within the Trunk Formula Method. Rather, it should be determined in the reconciliation process. After the appraising arborist has run several methods or approaches, he may then use those amounts to justify his final opinion. Rather than "fixing" the final value within the Trunk Formula Method, the adjustment should be placed after it has been run. This potential solution would still allow appraisers to adjust tree values downward, but the adjustment would take place in the reconciliation phase, and not within the framework of TFM

Conclusion

The graph (**Fig. 5**) compares the three formulas. The standard Trunk Area (TA) calculation without ATA becomes very large for diameters greater than 50 inches. The ATA formula eventually slopes down to 0 for very large trunk diameters. The $MTA_{Logistic}$ formula reaches an asymptote at an arbitrary maximum value. Finally, the MTA_{Ln} formula neither slopes downward nor reaches a maximum value and is derived by projecting a curve from the point at which the calculation switches from standard area to the Modified Trunk Area.

Ultimately, the decision between which alternative is

Figure 4. Graphic representation of the four curves discussed in this article showing the relationship of the diameter of a tree to its base price. Trunk Area (blue) is the unadjusted relationship of tree diameter to base price. Adjusted Trunk Area (red) is the currently published method of adjusting tree size. Modified Trunk Area Logistic (green) is the proposed alternative that has a maximum value. Modified Trunk Area Natural Log (purple) is the proposed alternative that has no maximum value but moderates its incremental rate of increase for larger trees. Note that the base price for the ATA method is negative on very large trees.



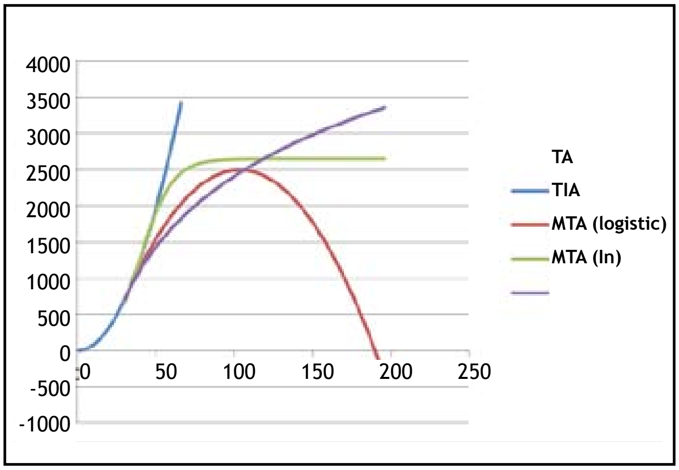


Figure 5: Graph comparing TA, ATA, $MTA_{Logistic}$, and MTA^{Ln}. The x-axis is the diameter measurement in inches, and the y-axis is the resultant area calculation in square inches.

better becomes a personal value decision. Should trees have a maximum value based on size? Or should trees continue to increase in value as they increase in size, regardless of how large they are? These questions must be answered by practitioners before the coefficients can be calculated and the formula can be justified.

Within the existing framework set by the CTLA, there is room to improve their concept of ATA. The function used in the formula may be changed to better model the intended behavior of the curve, or the ATA formula can be eliminated altogether. It seems as though the CTLA is moving toward the latter option.

Literature cited

CTLA. 2000. Council of Tree and Landscape Appraisers. Guide for Plant Appraisal, 9th ed. International Society of Arboriculture, Champaign, IL.

Watson, G. 2001. A study of CTLA Formula Values. J. Arbor. 27(6): 289-297.

Watson, G. 2002. Comparing formula methods of tree appraisal. J Arbor. 28(1): 11-18.

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